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WINCHESTER COLLEGE

# Entrance Examination 

## Mathematics

2020

Time allowed: 1 hour 30 minutes

Total marks: 100

## CALCULATORS ARE NOT ALLOWED.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams, but use a pen for everything else. You should show all your working so that credit may be given for partly correct answers.

Do not be discouraged if you do not finish. If you get more than 50 marks you will have done well.

1. Complete:
$\left.\begin{array}{l|l|l|l}\text { a) } 13^{2}= & \text { b) } \sqrt[3]{8000}= & \text { [1] } \\ \text { [1] }\end{array}\right]$
[1]
[1]
2. a) Find $\frac{4}{17}$ of $£ 510$
b) Find $74 \%$ of $£ 50$
c) Find 0.22 of $£ 400400$
d) Evaluate $5 \times 11 \times 2 \times 2 \times 17 \times 5$
e) Evaluate $20 \%$ of $30 \%$ of $40 \%$ of $£ 35000$
f) Evaluate $0.54 \times 321-22.1 \times 5.4$
3. Find in the simplest form:

4. The diagram below shows a hexagon whose vertices lie on a circle with centre O .
a) Explain why the shaded triangle is isosceles.
b) Find $a$.

c) The diagram shows two isosceles triangles inside a parallelogram. Find $x$ and $y$.

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5. a) $a=-3, b=5$ and $c=-12$.
b) $2(x-1)+5(x+1)-(x-3)=66$. Find $x$.

Evaluate $3 a^{3}-5 b+c^{2}$.
c) $\sqrt{\sqrt{y}+0.25}=1.5$. Find $y$.
d) $\frac{4444}{z^{3}+17}=101$. Find $z$.
6. a) The sum of five consecutive numbers is 155 . Find the largest of these numbers.
b) Put in ascending order:

$$
3 \sqrt{11}, \sqrt{101}, 7 \sqrt{2}, 10
$$

7. a) In the diagram below the ratio of the external angles, $x: y: z=4: 3: 2$. Find the ratio of the interior angles $a: b: c$, in its simplest form.
b) Carol, Diane and Eliza share $£ 210$ between them. The ratio of Carol’s money to Diane’s money is $2: 3$. The ratio of Diane's money to Eliza's money is $4: 5$. How much money did Carol get?
8. Xavier has been asked by his teacher to count the positive factors of 12 . He writes down the factors $1,2,3,4,6$ and 12 , and then gives his teacher the correct answer six. His teacher then asks him to find the number of factors of 216. She gives him the incomplete table below to help him.

Note that $216=2^{3} \times 3^{3}$.
a) Complete the table below by putting the remaining factors of 216 in the empty squares.
b) In the table, circle the factors of 216 that are square numbers.

c) How many factors of 25000 are there, and how many of these factors are square numbers?
9. Look at the sequence of fractions below:

$$
\frac{1}{1+\frac{2}{1}}, \frac{2}{1+\frac{3}{2}}, \frac{3}{1+\frac{4}{3}}, \frac{4}{1+\frac{5}{4}}, \cdots
$$

a) The first term simplifies to $\frac{1}{3}$. Write the first five terms of the sequence in simplified form.
b) Find the twentieth term, in the form $\frac{a}{b}$.
c) Look at the sequence of fractions below.

$$
\frac{1}{1+\frac{1}{1+\frac{2}{1}}}, \frac{1}{1+\frac{2}{1+\frac{3}{2}}}, \frac{1}{1+\frac{3}{1+\frac{4}{3}}}, \frac{1}{1+\frac{4}{1+\frac{5}{4}}}, \cdots
$$

Write the $99^{\text {th }}$ term of this sequence as a decimal.
10. a) Alan starts to walk up an upward-moving escalator of length 90 m at a constant speed, starting from the bottom. He gets to the top after thirty seconds. Had the escalator not been moving, Alan would have got to the top after fifty seconds. What is the speed of the escalator, assuming that it is constant?
b) Brenda starts to walk up a different upward-moving escalator of length 40 m at a constant speed, starting from the bottom. She gets to the top after twenty seconds. She then walks down the escalator, taking a further fifty seconds to reach the bottom. What is the speed of the escalator, assuming that it is constant? (Assume furthermore that Brenda always walks at the same speed.)
11. The surface area of a sphere is $4 \pi r^{2}$, where $r$ is the radius of the sphere.

The curved surface area of a cylinder is $2 \pi r h$, where $r$ is the radius of the cylinder and $h$ is the height.
a) In the diagram below, a solid has been formed by joining a hemisphere to a cylinder of equal radius. The height of the cylinder is equal to its diameter.

The total surface area of the solid (including the base) is $k \pi r^{2}$. Find $k$.

b) A sphere of radius 30 cm is cut into six identical segments. What is the total surface area of one segment? (Give your answer in terms of $\pi$.)
c) The solid shown below is one eighth of a sphere (so three faces are quarter-circles). What fraction of the total surface area of the solid is curved?

d) A number of identical spherical tennis balls fit exactly inside a cylindrical tube. The surface area of the tube (including its two ends) is $10 \%$ more than the total surface area of the tennis balls. How many tennis balls are there? (The diagram is not to scale.)
12. a) The diagram below shows a (dotted) cuboid which just fits inside a pyramid. The apex of the pyramid lies directly above the centre of its horizontal square base. The height of the cuboid is half the height of the pyramid. What fraction of the volume of the pyramid is the volume of the cuboid?
(The volume of a pyramid is equal to $\frac{1}{3} \times$ base area $\times$ height .)

b) The diagram below shows an octahedron. M is the point in the middle of the horizontal square BCDE. The points $\mathrm{A}, \mathrm{M}$ and F lie on the same vertical line. The length of the line AB is 20 and the length of the dotted diagonal BD is 24 . The lines AB and BF are at right angles to each other.

Find the volume of the octahedron.


(END OF PAPER)

