

Name

School



WINCHESTER
COLLEGE

Entrance Examination

Mathematics

Tuesday 7 May 2019

Time allowed: 1 hour 30 minutes

Total marks: 100

Solutions

CALCULATORS ARE NOT ALLOWED.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams. You should show all your working so that credit may be given for partly correct answers.

Do not be discouraged if you do not finish. If you get more than 60 marks, you will have done well.

1.	<p>Complete:</p> <p>a) $7 \times 13 = 91$</p>	<p>b) $\sqrt{400} = 20$</p>	<p>[1] [1]</p>
	<p>c) $4321 + (9753 - 4321) = 9753$</p>	<p>d) $1 \times 111 + 2 \times 111 + 4 \times 111 = 777$</p>	<p>[1] [1]</p>
	<p>e) $\frac{125 + 150 + 175}{3} = 150$</p>	<p>f) $5 \div \frac{1}{7} = 35$</p>	<p>[1] [1]</p>
	<p>g) $\sqrt[3]{125} = 5$</p>	<p>h) $\frac{224466}{22} = 10203$</p>	<p>[1] [1]</p>

2.	<p>a) Find $\frac{11}{12}$ of £108</p> $= 11 \times (\pounds 108 \div 12)$ $= 11 \times \pounds 9$ $= \pounds 99$	<p>b) Find 68% of £25</p> $= \pounds (68 \div 4)$ $= \pounds 17$	[1] [1]
	<p>c) Find 0.05 of £440</p> $= \pounds 440 \div 20$ $= \pounds 22$	<p>d) Evaluate $8 - 6 \times (-2) - 6 \times 18 \div (-3)$</p> $= 8 + 12 + 36$ $= 56$	[1] [2]
	<p>e) Evaluate $1010 + 1001 \times 1009 - 1000 \times 1009$</p> $= 1010 + 1009$ $= 2019$	<p>f) Evaluate $\frac{(1 \times 2 \times 3 \times 4 \times 5 \times 6)^2}{(1 \times 2 \times 3 \times 4)^2}$</p> $= (5 \times 6)^2$ $= 900$	[2] [3]

3. Find in the simplest form:

$$\begin{aligned} \text{a) } \frac{8}{11} - \frac{2}{33} &= \frac{24}{33} - \frac{2}{33} \\ &= \frac{22}{33} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 7\frac{1}{5} \div 2\frac{2}{5} &= \frac{36}{5} \div \frac{12}{5} \\ &= \frac{36}{12} \\ &= 3 \end{aligned}$$

[2]
[2]

$$\begin{aligned} \text{c) } \sqrt{\frac{22}{5} \times \frac{165}{6}} &= \sqrt{\frac{11}{5} \times \frac{165}{3}} \\ &= \sqrt{\frac{11}{5} \times \frac{55}{1}} \\ &= \sqrt{\frac{11}{1} \times \frac{11}{1}} \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{5}{2 - \frac{1}{1 - \frac{1}{3}}} &= \frac{5}{2 - \frac{1}{\frac{2}{3}}} \\ &= \frac{5}{2 - \frac{3}{2}} \\ &= 5 \div \frac{1}{2} \\ &= 10 \end{aligned}$$

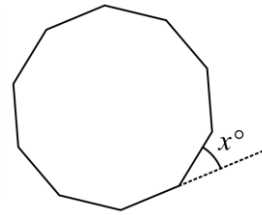
[2]
[3]

<p>4.</p>	<p>a) $p = 3$, $q = 12$ and $r = -5$.</p> <p>Evaluate $p\sqrt{q^2 + r^2}$.</p> $= 3 \times \sqrt{144 + 25}$ $= 3 \times \sqrt{169}$ $= 3 \times 13$ $= 39$	<p>b) $2(x+3) - (x-5) + 4(x+2) = 119$. Find x.</p> $2x + 6 - x + 5 + 4x + 8 = 119$ $5x = 100$ $x = 20$	<p>[2] [2]</p>
	<p>c) $\frac{c^3 - 14}{5} = 10$. Find c.</p> $c^3 - 14 = 50$ $c^3 = 64$ $c = 4$	<p>d) $\frac{72}{y - 21} = 8$. Find y.</p> $\frac{72}{8} = y - 21$ $y - 21 = 9$ $y = 30$	<p>[2] [2]</p>

5.

a) The diagram below shows a regular decagon (ten equal sides with ten equal interior angles). Write down the value of x .

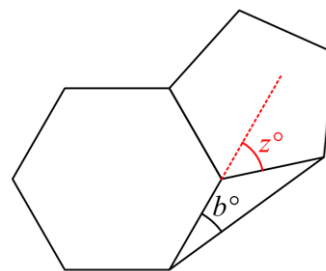
$$x = 36$$



[1]

b) The diagram below shows a regular hexagon and a regular pentagon (which share a side). Find the value of b .

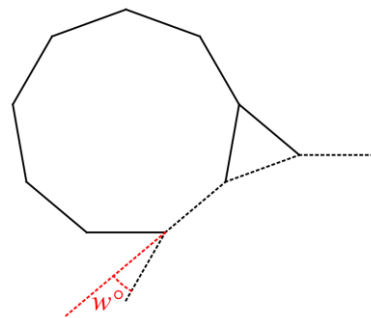
$$\begin{aligned} z &= 120 + 108 - 180 \\ &= 48 \\ b &= z \div 2 \\ &= 24 \end{aligned}$$



[3]

c) The diagram below shows a regular nonagon (nine sides), an equilateral triangle and part of a regular polygon (dotted) with n sides. Find n .

$$\begin{aligned} w &= 60 - \frac{360}{9} \\ &= 20 \\ n &= \frac{360}{20} \\ &= 18 \end{aligned}$$

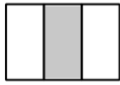


[3]

<p>6.</p>	<p>a) Find the value of $\frac{16^3}{2^9}$.</p> $= \frac{(2^4)^3}{2^9}$ $= \frac{2^{12}}{2^9}$ $= 2^3$ $= 8$	<p>b) $10^a \times 10^b = 10^7$. $10^a \div 10^b = 10^3$.</p> <p>Find the value of ab.</p> $a + b = 7$ $a - b = 3$ $a = 5$ $b = 2$ $ab = 10$	<p>[2] [2]</p>
	<p>c) By writing each number as a product of prime factors, evaluate $\sqrt{15 \times 21 \times 35}$.</p> $= \sqrt{3 \times 5 \times 3 \times 7 \times 5 \times 7}$ $= \sqrt{3^2 \times 5^2 \times 7^2}$ $= 3 \times 5 \times 7$ $= 105$	<p>d) By writing 2020 as a product of prime factors, find a three-digit number n for which $2020n$ is a square number.</p> $2020 = 2^2 \times 5 \times 101$ $n = 5 \times 101$ $= 505$	<p>[2] [3]</p>

7. a)

Strip Pattern 1



Strip Pattern 2



Strip Pattern 3



Strip Pattern number	1	2	3	4	5	6	n
Fraction of strip that is shaded	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$	$\frac{5}{7}$	$\frac{6}{8}$	$\frac{n}{n+2}$

(i) Complete the table above.

[2]

(ii) Strip pattern k is 90% shaded. Find k .

[2]

$$\begin{aligned} \frac{k}{k+2} &= \frac{9}{10} \\ 10k &= 9k + 18 \\ k &= 18 \end{aligned}$$

b) Complete the table below (a match is one side of a small square).

[3]

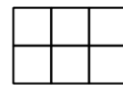
Rectangle Pattern 1



Rectangle Pattern 2



Rectangle Pattern 3

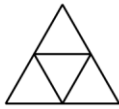


Rectangle Pattern number	1	2	3	4	11	n
Number of matches	7	12	17	22	57	$5n + 2$

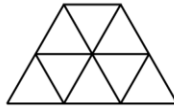
c) Find a formula for the number of matches in trapezium pattern n (a match is one side of a small equilateral triangle).

[2]

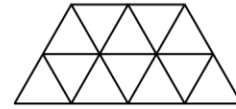
Trapezium Pattern 1



Trapezium Pattern 2



Trapezium Pattern 3



$$= 7n + 2$$

d) Frank makes a rectangle pattern, using all the matches he has. He then rearranges these matches into a trapezium pattern and has no matches left. Find three possible values for the number of matches Frank started with.

[3]

$$5n + 2 = 7m + 2$$

$$5n = 7m$$

The number of matches is a multiple of 35, plus 2.

Three possible values are 37, 72 and 107

8.

a) Alice, Brenda and Clara share a pile of buttons in the ratio 7:12:21. Clara gets ten more buttons than Alice and Brenda put together. How many buttons were in the pile?

[2]

$$21 - (7 + 12) = 2$$

$$\begin{aligned} \text{Total} &= \frac{10}{2} \times (7 + 12 + 21) \\ &= 200 \end{aligned}$$

b) Derek, Ethan, Fynn and Gerald share a big pile of sweets. Derek, Ethan and Fynn now have sweets in the ratio 1:4:6, and Ethan, Fynn and Gerald have sweets in the ratio 6:9:2. Derek and Gerald have 35 sweets between them. How many sweets does Gerald have?

[3]

$$\begin{aligned} D : E : F : G \\ 3 : 12 : 18 : 4 \end{aligned}$$

Derek and Gerald have 35 sweets between them, so Gerald has $\frac{35}{7} \times 4 = 20$ sweets.

c) Harriet and Imogen share sweets in the ratio 1:2. Imogen gives Harriet 42 sweets. The sweets are now shared between Harriet and Imogen in the ratio 5:4. How many sweets did they share?

[4]

$$1:2 = 3:6.$$

Suppose they have nine sweets.

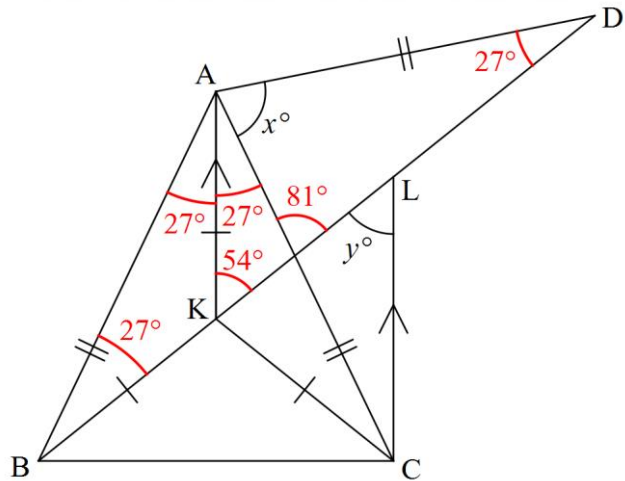
3:6 becomes 5:4 if Imogen gives Harriet 2 sweets.

So they shared $\frac{42}{2} \times 9 = 189$ sweets.

9.

a) In the diagram below $AB = AC = AD$, $KA = KB = KC$, KA is parallel to CL , and BKD is a straight line.

Angle $BAC = 54^\circ$. Find x and y .



[5]

$$x = 72$$

$$y = 54$$

b) $ABCD$ is a rhombus and KLC is an equilateral triangle. The lengths of the sides of KLC are the same as the lengths of the sides of $ABCD$. Find z .

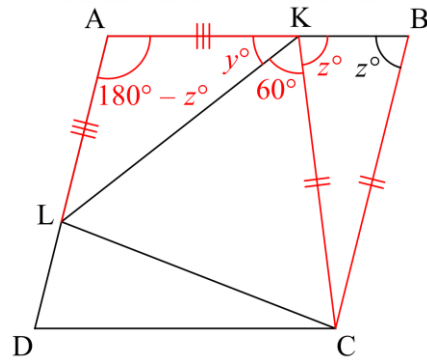
[5]

$$y = \frac{z}{2}$$

$$z + 60 + y = 180$$

$$z + \frac{z}{2} = 120$$

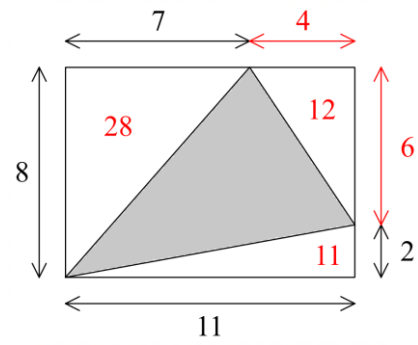
$$z = 80$$



10. a) Find the shaded area in the diagram on the right.

[3]

$$\begin{aligned} \text{Shaded area} &= 88 - 28 - 12 - 11 \\ &= 37 \end{aligned}$$

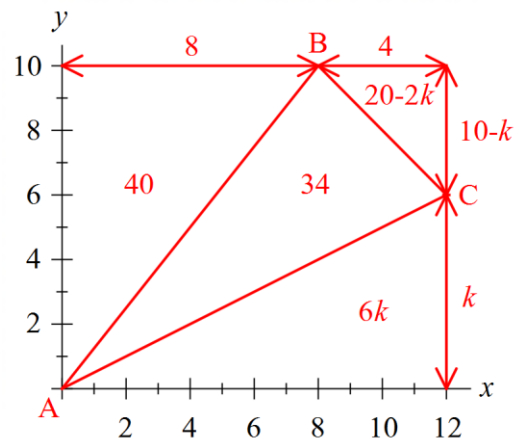


- b) The vertices of a triangle ABC are A(0, 0), B(8, 10) and C(12, k).

The area of the triangle ABC is 34.

- (i) Plot the points A and B on the diagram.
(ii) Find the value of k , given that $0 < k < 10$.

$$\begin{aligned} 40 + 34 + (20 - 2k) + 6k &= 120 \\ 94 + 4k &= 120 \\ 4k &= 26 \\ k &= 6.5 \end{aligned}$$



[6]

11. The diagram below shows a house comprising a cuboid and a pyramid. EDZ is a straight line. WX = 12, EB = 17, BC = 9 and BX = 10. Find the volume of the house.

[9]

(The volume of a pyramid is equal to $\frac{1}{3} \times \text{base area} \times \text{height}$.)

$$\begin{aligned}
 DB &= \sqrt{12^2 + 9^2} \\
 &= 15 \\
 ED &= \sqrt{17^2 - 15^2} \\
 &= 8 \\
 \text{Volume of cuboid} &= 12 \times 9 \times 10 \\
 &= 1080 \\
 \text{Volume of pyramid} &= \frac{1}{3} \times 12 \times 9 \times 8 \\
 &= 288 \\
 \text{Volume of house} &= 1080 + 288 \\
 &= 1368
 \end{aligned}$$

