Name ……………………….…...……………………

School ……………………….…...……………………

WINCHESTER
COLLEGE

Entrance Examination

Solutions

Mathematics

Tuesday 1 May 2018

Time allowed: 1 hour 30 minutes

Total marks: 120

CALCULATORS ARE NOT ALLOWED.

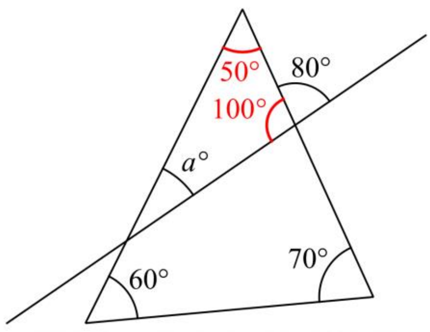
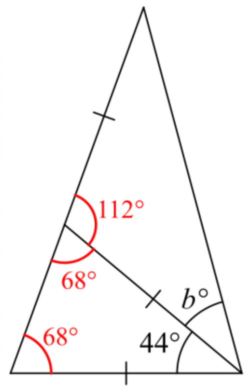
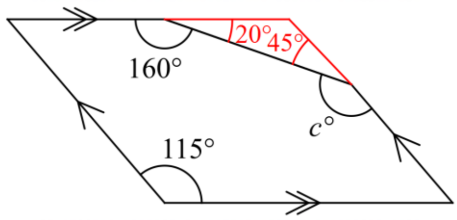
Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams. You should show all your working so that credit may be given for partly correct answers.

Do not be discouraged if you do not finish. If you get more than 60 marks, you will have done well.

1.	Complete: a) $9 \times 12 = 108$	b) $1000 - 999 + 2000 - 1998 + 3000 - 2997 = 6$	[1] [1]
	c) $1234 - (1234 - 7) = 7$	d) $(\sqrt{169})^2 = 169$	[1] [1]
	e) $\frac{1356 + 1358}{2} = 1357$	f) $1 \div \frac{1}{42} = 42$	[1] [1]
	g) $\sqrt{0.0025} = 0.05$	h) $\frac{3333}{101} = 33$	[2] [2]

2.	$a=5$, $b=10$ and $c=-2$. Find the value of:		
	a) $(a+3)(b+1)=88$	b) $a-b+c=-7$	[1] [1]
	c) $\frac{3690b}{123a}=60$	d) $\sqrt{b-3c}=4$	[2] [2]
	e) $b^2c^2=400$	f) $b^3+7b^2+2b+9=1729$	[2] [2]

3.	Find in the simplest form: a) $\frac{7}{9}+\frac{1}{18}=\frac{14}{18}+\frac{1}{18}$ $=\frac{15}{18}$ $=\frac{5}{6}$	b) $1\frac{1}{7}\times1\frac{3}{4}=\frac{8}{7}\times\frac{7}{4}$ $=2$	[2] [2]
	c) $\frac{18}{5}\div\frac{3}{10}=\frac{18}{5}\times\frac{10}{3}$ $=6\times2$ $=12$	d) $\dfrac{\frac{1}{3}+\frac{2}{9}+\frac{5}{27}}{\frac{1}{3}+\frac{2}{9}-\frac{5}{27}}=\dfrac{\frac{9}{27}+\frac{6}{27}+\frac{5}{27}}{\frac{9}{27}+\frac{6}{27}-\frac{5}{27}}$ $=\frac{20}{10}$ $=2$	[2] [3]

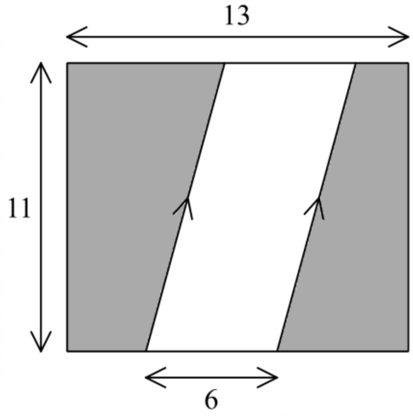
4.	a) Find a. $a = 30$  	[2]
	b) Find b. $b = 34$  	[3]
	c) Find c. $c = 135$  	[3]

5.	a) Evaluate $\dfrac{0.2\times 0.03}{0.004}$. $=\dfrac{2\times 3}{4}$ $=\dfrac{3}{2}$	b) Express $\dfrac{33\times 55}{44\times 66}$ in its simplest form. $=\dfrac{3\times 5}{4\times 6}$ $=\dfrac{5}{8}$	[2] [2]
	c) a is 25% more than 220, and b is 20% less than a. Find b. $b=220\times 1.25\times 0.8$ $=220$	d) Evaluate $\sqrt[3]{11\times 22\times 44}$. $=\sqrt[3]{11^3\times 2^3}$ $=11\times 2$ $=22$	[2] [2]
	e) Evaluate $\sqrt{3\frac{1}{16}}$. $=\sqrt{\dfrac{49}{16}}$ $=\dfrac{7}{4}$	f) Evaluate $\dfrac{(-2)^{12}}{2^7}$. $=2^5$ $=32$	[3] [3]

6. a) Find the shaded area in the rectangle below. [3]

$= (13 - 6) \times 11$

$= 77$

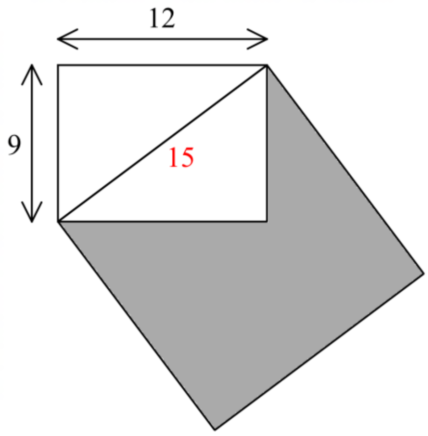


b) In the diagram below, the diagonal of a rectangle is the side of a square. Find the shaded area. [4]

$= 15^2 - \dfrac{12 \times 9}{2}$

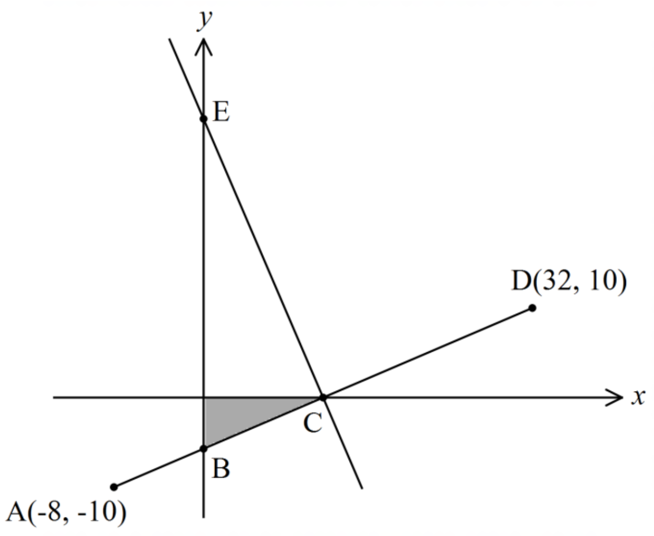
$= 225 - 54$

$= 171$



7.	a) $5a-1=\frac{4a+13}{2}$. Find a. $10a-2=4a+13$ $6a=15$ $a=\frac{5}{2}$	b) $\sqrt{200-\frac{52}{b}}=14$. Find b. $200-\frac{52}{b}=196$ $\frac{52}{b}=4$ $b=13$	[2] [2]
	c) $\frac{144}{6+\frac{36}{c}}=12$. Find c. $6+\frac{36}{c}=12$ $\frac{36}{c}=6$ $c=6$	d) $\frac{-27}{d}=\frac{d^2}{8}$. Find d. $d^3=-27\times 8$ $d=-3\times 2$ $=-6$	[3] [4]

8. In the diagram below, A is the point with coordinates (-8, -10) and D is the point with coordinates (32, 10). The straight line from A to D crosses the *y*-axis at B and the *x*-axis at C, and is perpendicular to a straight line which passes through C and crosses the *y*-axis at E.



a) Find the *x*-coordinate of the point C. [2]

Gradient $= 0.5$

$C(12,\ 0)$

(x-coordinate is 12.)

b) Find the shaded area. [4]

$B(0,-6)$

Shaded area $= 0.5 \times 12 \times 6$

$= 36$

c) Find the *y*-coordinate of the point E. [3]

Gradient of perpendicular $= -2$

$E(0, 24)$

(y-coordinate is 24.)

9.	a) In a sequence of ten numbers, the first number is 1, the second number is 1 and every number after that is the sum of the two numbers before it: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. Put the two missing numbers on the dotted lines.	[1]
	b) In a sequence of six numbers, every number after the first two numbers is the sum of the two numbers before it. The fifth number is 17 and the sixth number is 27: 4, 3, 7, 10, 17, 27. Put the four missing numbers on the dotted lines.	[2]
	c) In another sequence, the first number is 5, the second number is c, and every number after that is the sum of the two numbers before it. The ninth number is 107. Find c. (*Hint*: the sequence is 5, c, $5+c$, $5+2c$, $10+3c$, … .)	[3]
	The sequence is 5, c, $5+c$, $5+2c$, $10+3c$, $15+5c$, $25+8c$, $40+13c$, $65+21c$ … . $65+21c=107$ $21c=42$ $c=2$	

d) In yet another sequence, the first number is *x*, the second number is *y*, and every number after that is the sum of the two numbers before it. The fifth number is 48 and the eighth number is 202. Find *x* and *y*. [5]

The sequence is $x, y, x+y, x+2y, 2x+3y, 3x+5y, 5x+8y, 8x+13y \ \ldots$.

$$2x+3y=48$$

$$8x+13y=202$$

$$8x+12y=192$$

$$y=10$$

$$x=9$$

10. a) Adrian, Brian and Chris share a big pile of sweets in the ratio $6:11:13$. Adrian and Brian together get 28 more sweets than Chris. How many sweets do they have altogether? [2]

$6+11-13=4$

$\frac{28}{4}=7$

$7\times(6+11+13)=210$

b) Daniel, Edward and Fynn share a big pile of sweets in the ratio $4:3:11$, and a bigger pile of sweets in the ratio $4:9:17$. They now have sweets in the ratio $2:3:x$. Find *x*. [3]

$2:3:x=4:6:2x$

6 is halfway between 3 and 9, so 2*x* is halfway between 11 and 17.

$2x=14$

$x=7$

c) George, Harry and Isaac share a big pile of sweets in the ratio $15:8:2$. Harry, Isaac and Jake share another big pile of sweets in the ratio $1:4:10$. Now Harry and Isaac have the same number of sweets, and Jake has 45 more sweets than George. How many sweets do the four boys have altogether? [5]

$1:4:10=2:8:20$

G	H	I	J	
15	8	2		
	2	8	20	
15	10	10	20	sum = 55

$20-15=5$

$\frac{45}{5}=9$

$9\times55=495$

11.	$xyz = 64$, where x, y and z are positive integers. One solution is $x = 1$, $y = 2$ and $z = 32$. Another is $x = 2$, $y = 1$ and $z = 32$. How many solutions are there in total? (You do not need to write all the solutions down, but you do need to show clearly how you have counted them.)	[9]

$64 = 2^6$, so x, y and z must all be powers of 2 and these powers must add up to 6:

$x = 2^6,\ y = 2^0,\ z = 2^0$; $x = 2^0,\ y = 2^6,\ z = 2^0$; $x = 2^0,\ y = 2^0,\ z = 2^6$ (3 solutions)

$x = 2^5,\ y = 2^1,\ z = 2^0$ (6 solutions)

$x = 2^4,\ y = 2^2,\ z = 2^0$ (6 solutions)

$x = 2^4,\ y = 2^1,\ z = 2^1$ (3 solutions)

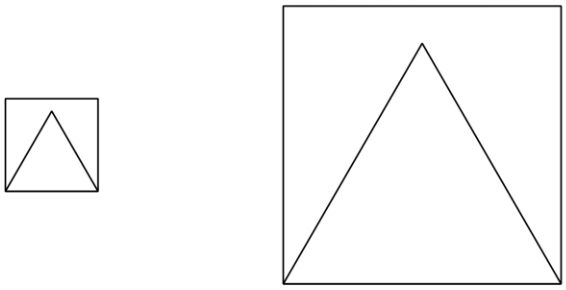
$x = 2^3,\ y = 2^3,\ z = 2^0$ (3 solutions)

$x = 2^3,\ y = 2^2,\ z = 2^1$ (6 solutions)

$x = 2^2,\ y = 2^2,\ z = 2^2$ (1 solutions)

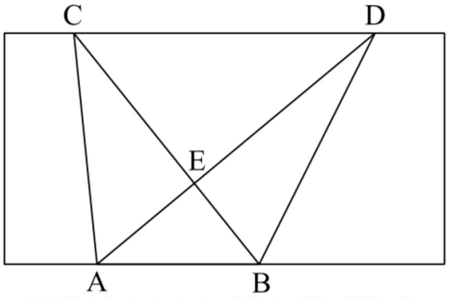
28 solutions in all.

12.	a) In the diagram below, the length of the side of the big square is three times the length of the side of the small square. The area of the small equilateral triangle is 11. What is the area of the big equilateral triangle?	[2]



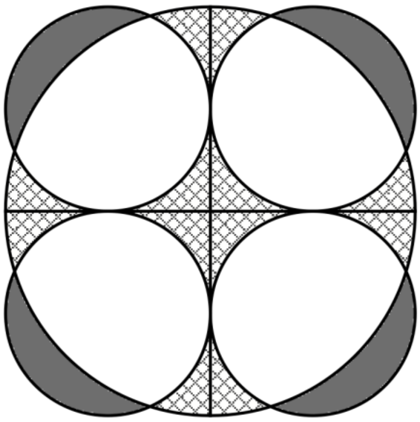
$11 \times 3^2 = 99$

b) In the diagram below, the straight lines AD and BC intersect at E. The area of triangle BED is 24. Find the area of triangle AEC. *Explain your reasoning carefully*. [3]



Triangles ABC and ABD have the same base and the same height.
So triangles ABC and ABD have the same area.
These areas include the common area ABE.
So the areas of AEC and BED are the same.
So the area of AEC is 24.

c) In the diagram below, the light shaded area is equal to 44. Find the dark shaded area. *Explain your reasoning carefully*. [7]



The area of the large circle is four times the area of a small circle.
(Because it has twice the radius.)
So the area of the large circle is equal to the sum of the areas of the four small circles:

light shaded area + white area = white area + dark shaded area

light shaded area = dark shaded area

dark shaded area = 44

(END OF PAPER)