Name ....

WINCHESTER

COLLEGE

ELECTION

SOLUTIONS

Mathematics 2

Wednesday 8 May 2019

Time allowed: 1 hour 30 minutes

Total marks: 100

Calculators are not allowed.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams.

Work carefully, and *do not be discouraged if you do not finish*.

You should show your working so that credit may be given for partly correct answers.

1.

a) Evaluate $28 \div \frac{7}{3}$. [1]

$= 28 \times 3/7$

$= 12$

b) Evaluate $(3\sqrt{11})^2 + 1$. [1]

$= 99 + 1$

$= 100$

c) Evaluate $\sqrt{1.96}$. [1]

$= 1.4$

d) Evaluate $\frac{0.09 \times 0.028}{0.00006}$. [1]

$= \frac{9 \times 28}{6}$

$= \frac{3 \times 28}{2}$

$= 3 \times 14$

$= 42$

e) What fraction is halfway between $\frac{1}{4}$ and $\frac{1}{5}$? [2]

$1/4 = 5/20 = 10/40$

$1/5 = 4/20 = 8/40$

$9/40$ is halfway between

f) Find 20% of 40% of 80% of 5000. [2]

$2/10 \times 4/10 \times 8/10 \times 5000$

$= 2 \times 4 \times 8 \times 5$

$= 320$

2. Solve:

a) $4(7-x)-5(2x-1)=12$

$28-4x-10x+5=12$

$21=14x$

$x=3/2$

b) $\left(\frac{x-1}{11}\right)^3=8$ [2] [2]

$\frac{x-1}{11}=2$

$x-1=22$

$x=23$

c) $\frac{2}{x-5}=\frac{3}{x+1}$

$2(x+1)=3(x-5)$

$2x+2=3x-15$

$x=17$

d) $\frac{77}{15-\frac{8}{7-x}}=7$ [2] [2]

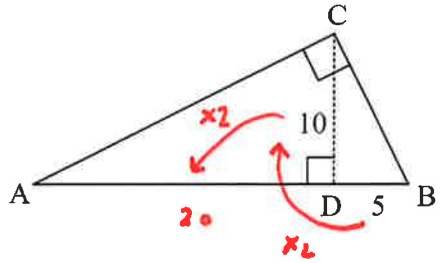
$15-\frac{8}{7-x}=11$

$\frac{8}{7-x}=4$

$7-x=2$

$x=5$

3. a) In the diagram below, $CD = 10$ and $BD = 5$. Find the area of the triangle ABC. [3]



Area $= \frac{1}{2} \times 25 \times 10$

$= 125$

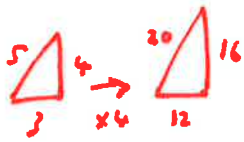
b) In the diagram below, $AD = 12$, $CD = 9$ and $AB = 20$. Find the area of the triangle ABC. [4]

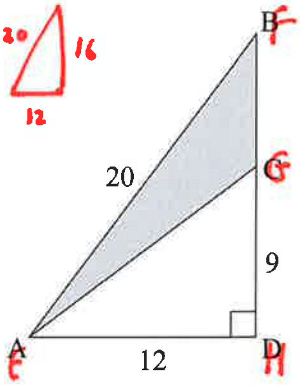
EH GH EF EFG

$BD = 16$, by Pythagoras

$BC = 16 - 9$

$= 7$





Area $= \frac{1}{2} \times 7 \times 12$

$= 42$

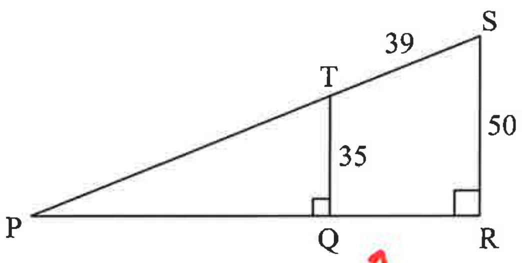
c) In the diagram below, $TQ = 35$, $SR = 50$ and $TS = 39$. Find the area of the triangle PRS. [5]

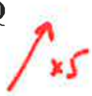
$35 : 50 = 7 : 10$

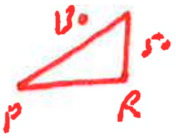
So $PS : TS = 10 : 3$

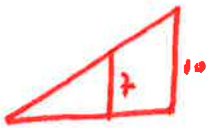
So $PS = \frac{10}{3} \times 39$

$= 130$









$PR = 120$, by Pythagoras

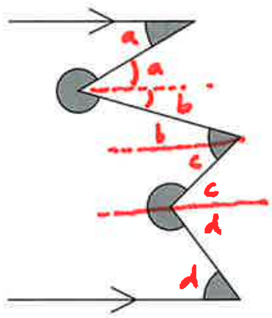
Area $= \frac{1}{2} \times 120 \times 50$

$= 3000$

4. a) In the diagram below, two of the six lines are parallel. Find the sum of the shaded angles. [2]

Angles sum to $2 \times 360°$

$= 720°$



b) In the diagram below, there are six straight lines. Find the sum of the shaded angles. [3]

$a + b + c = 180°$

Sum of shaded angles

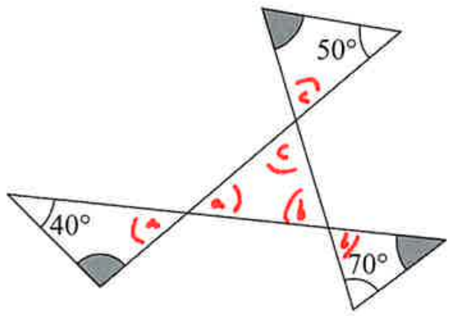
$+ (40° + 50° + 60°)$

$+ a + b + c$

$= 3 \times 180°$

Sum of shaded angles $+ 160° = 2 \times 180°$

Sum $= 200°$



c) In the diagram below, there are five straight lines. Find the sum of the shaded angles. [3]

(*Hint*: you might consider the sum of the ~~marked~~ arrowed angles.)

Sum of marked angles $= 360°$

$a + b + c + d + e = 360°$ Similarly

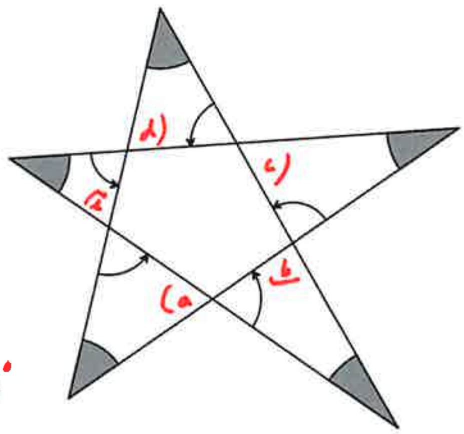
Sum of shaded angles

$+$ Sum of marked angles

$+ (a + b + c + d + e) = 5 \times 180°$

Sum of shaded angles $= 5 \times 180° - 360° - 360°$

$= 180°$



5.	a) Multiply out $(a-b)(a+b)$.	[1]

$(a-b)(a+b) = a^2 - b^2$

	b) Find all the solutions of the equation $a^2 - b^2 = 99$, where a and b are (positive) whole numbers.	[6]

$(a-b)(a+b) = 3 \times 3 \times 11$

$= 1 \times 99$

$a-b = 1$
$a+b = 99$
$2a = 100$; $a = 50$, $b = 49$

or 3×33

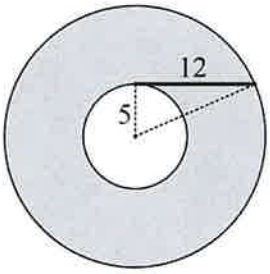
$a-b = 3$
$a+b = 33$
$2a = 36$; $a = 18$, $b = 15$

or 9×11

$a-b = 9$
$a+b = 11$
$2a = 20$; $a = 10$, $b = 1$

6. In all the diagrams below, the circles are concentric, radii are dotted and tangents are bold.

a) Find the shaded area in the diagram below. [2]



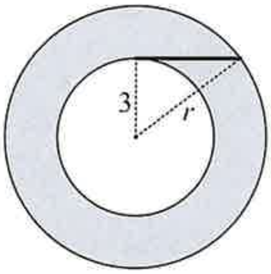
radius of large circle = 13

area = $\pi \times 13^2 - \pi \times 5^2$

$= \pi \times 12^2$

$= 144\pi$

b) The shaded area in the diagram below is 16π. Find r, the radius of the larger circle. [2]



area of large circle

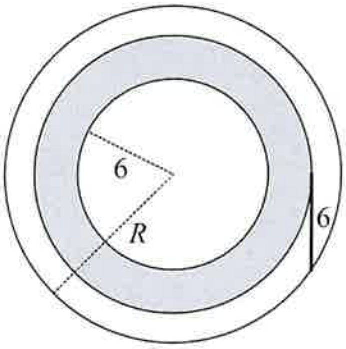
$= \pi \times 3^2 + 16\pi$

$= 25\pi$

$\pi r^2 = 25\pi$

$r = 5$

c) In the diagram below, the shaded area is 49π. Find R, the radius of the largest circle. [4]

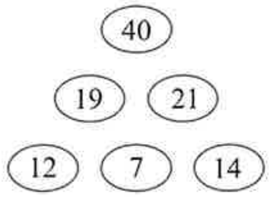


$R = \sqrt{6^2 + 49 + 6^2}$

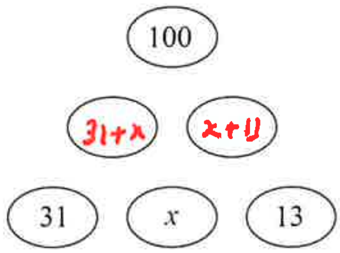
$= \sqrt{121}$

$= 11$

7. In a *plus-pyramid*, every number is a whole number greater than zero and is the sum of the two numbers below it. The diagram below gives an example.



a) The diagram below shows another plus-pyramid. Find x. [2]

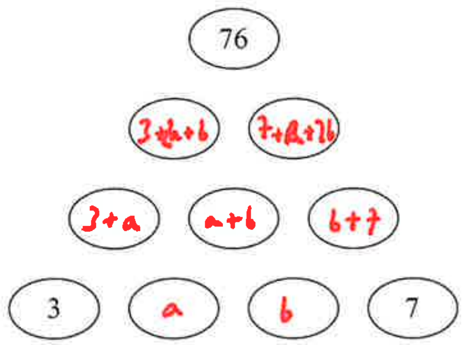


$2x + 44 = 100$

$2x = 56$

$x = 28$

b) How many different ways of completing the plus-pyramid below are there? [4]



$76 = 3 + 2a + b + 7 + a + 2b$

$= 10 + 3a + 3b$

$3a + 3b = 66$

$a + b = 22$

$a = 1, \; b = 21$

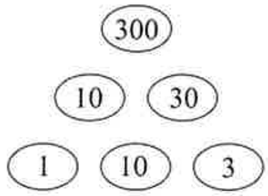
$a = 2, \; b = 20$

$\vdots$

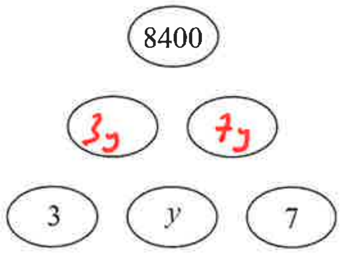
$a = 21, \; b = 1$

There are 21 ways.

In a *times-pyramid*, every number is a whole number greater than zero and is the product of the two numbers below it. The diagram below gives an example.



a) The diagram below shows another times-pyramid. Find y. [2]

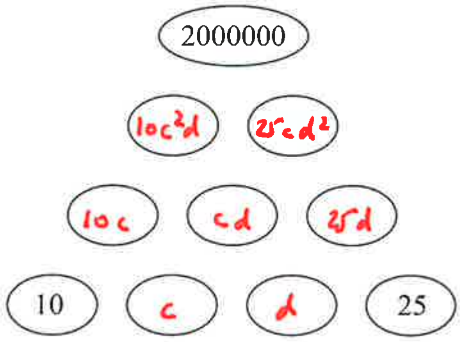


$21y^2 = 8400$

$y^2 = 400$

$y = 20$

b) How many different ways of completing the times-pyramid below are there? [5]



$2000000 = 10c^2d \times 25cd^2$

$= 250c^3d^3$

$8000 = c^3d^3$

$cd = 20$

$c = 1, \quad d = 20$

$c = 2, \quad d = 10$

$c = 4, \quad d = 5$

$c = 5, \quad d = 4$

$c = 10, \quad d = 2$

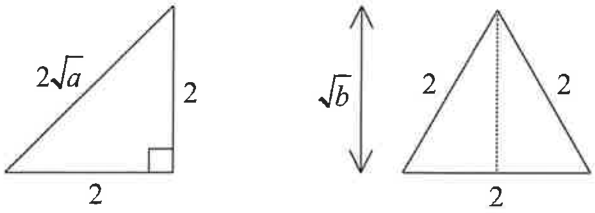
$c = 20, \quad d = 1$

There are 6 ways.

8. a) Find a and b. [2]

$a = 2$

$b = 3$

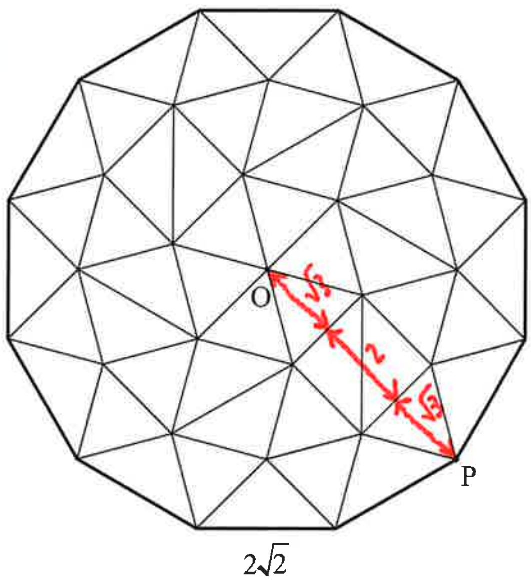


b) The diagram below shows a bold regular dodecagon (12 sides) divided into equilateral triangles (all the same size) and right-angled isosceles triangles (also all the same size). The length of a side of the dodecagon is $2\sqrt{2}$. Find the area of the dodecagon. Give your answer in the form $c + d\sqrt{3}$. [3]

24 △, each of area $\sqrt{3}$

24 ◺, each of area 2

Total area $= 48 + 24\sqrt{3}$



c) Find the distance OP. [1]

$OP = 2 + 2\sqrt{3}$

d) Find $(2+2\sqrt{3})^2$, in the form $a+b\sqrt{3}$. You might find the diagram below useful. [2]

	2	$2\sqrt{3}$
2	4	$4\sqrt{3}$
$2\sqrt{3}$	$4\sqrt{3}$	12

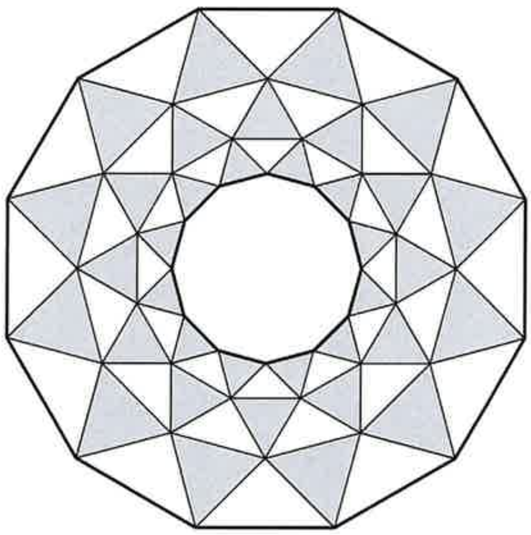
$(2+2\sqrt{3})^2 = 16 + 8\sqrt{3}$

e) Using the fact that the area of the circle through the vertices of the dodecagon has area $\pi(\mathrm{OP})^2$, find an approximation to π. Show your working clearly. [2]

$\pi\,(16 + 8\sqrt{3}) = 48 + 24\sqrt{3}$

$\pi = 3$

f) All the shaded triangles in the diagram below are equilateral. The area of the bold outer regular dodecagon is 40. Find the area of the bold innermost regular dodecagon. [4]



Outer ring of $12\Delta + 12\Delta$ is half the area of the outer regular dodecagon.

So the innermost regular dodecagon has area

$40 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= 5$

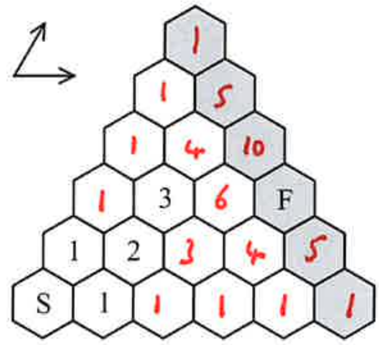
9. A bee starts in cell S and moves from cell to cell, always moving in the direction of one of the arrows (to the right, or up and to the right).

a) How many routes to cell F can the bee take? [3]

(*Hint*: one cell has a three in it because there are three routes to that cell.)

10

(Pascal's Δ)



b) How many routes to the shaded row can he take? [3]

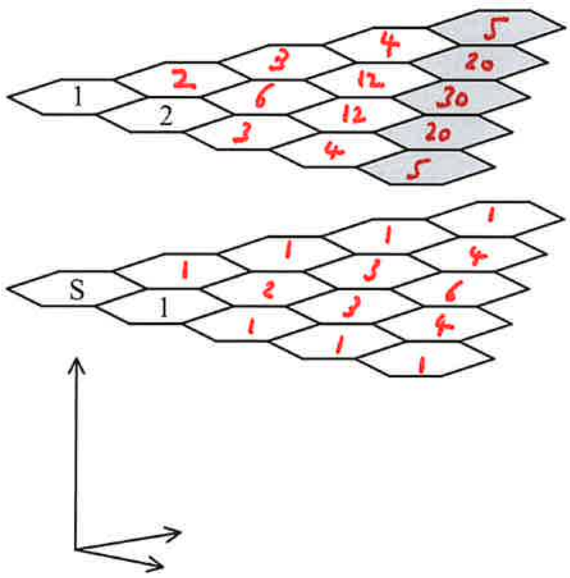
$2^5 = 32$

(or $1 + 5 + 10 + 10 + 5 + 1 = 32$)

c) The bee now navigates two layers of cells in the 3-D diagram below, always moving in the direction of one of the arrows. How many routes to shaded row can he take? [4]

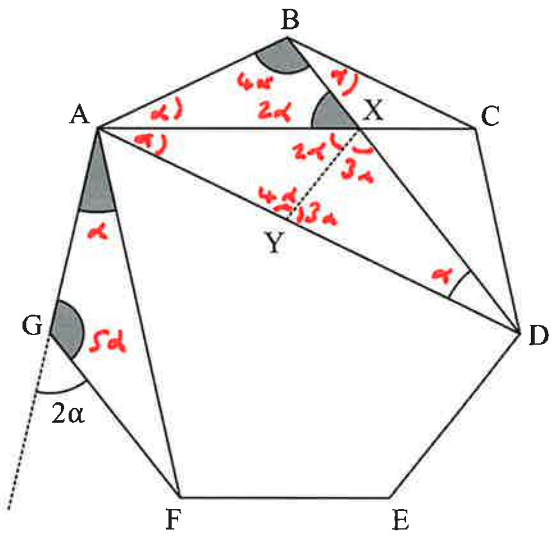
(One cell has a two in it because there are two routes to that cell.)

$2^4 \times 5 = 80$



10. The diagram shows a regular heptagon with external angle 2α. X lies on AC and BD.

a) Find the value of 7α. [1]



Sum of external angles

$= 7 \times 2\alpha$

$= 360^\circ$

So $7\alpha = 180^\circ$

b) Write the value of each of the shaded angles, in terms of α, on the diagram. Also prove that $\angle XDY = \alpha$. [6]

$\angle XDY = \angle DBC$ (Z-angle)

$= \alpha$

c) Triangle XYD is isosceles. Prove that $AB + AX = AD$. [5]

$AB = AY$ (ABX & AYX congruent)

$AX = XD$ (AXD isosceles)

$= YD$ (XDY isosceles)

So $AB + AX = AY + YD$

$= AD$

(END OF PAPER)