

Name



WINCHESTER
COLLEGE

ELECTION

SOLUTIONS

Mathematics 1

Monday 6 May 2019

Time allowed: 1 hour 30 minutes

Total marks: 100

Calculators are not allowed.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams.

Work carefully, and *do not be discouraged if you do not finish.*

You should show your working so that credit may be given for partly correct answers.

1. a) Find $\frac{11}{12}$ of £108

$$\begin{aligned} & \frac{11}{12} \times 108 \\ & = 11 \times 9 \\ & = 99 \end{aligned}$$

b) Find 68% of £25

$$\begin{aligned} & = \pounds (68 \div 4) \\ & = \pounds 17 \end{aligned}$$

[1]
[1]

c) Find 0.05 of £440

$$\begin{aligned} & = \frac{\pounds 440}{20} \\ & = \pounds 22 \end{aligned}$$

d) Evaluate $8 - 6 \times (-2) - 6 \times 18 \div (-3)$

$$\begin{aligned} & = 8 + 12 + 36 \\ & = 56 \end{aligned}$$

[1]
[2]

e) Evaluate $1010 + 1001 \times 1009 - 1000 \times 1009$

$$\begin{aligned} & = 1010 + 1019 \\ & = 2019 \end{aligned}$$

f) Evaluate $\frac{(1 \times 2 \times 3 \times 4 \times 5 \times 6)^2}{(1 \times 2 \times 3 \times 4)^2}$

$$\begin{aligned} & = (5 \times 6)^2 \\ & = 900 \end{aligned}$$

[2]
[3]

2. Find in the simplest form:

$$\begin{aligned} \text{a) } \frac{8}{11} - \frac{2}{33} &= \frac{24}{33} - \frac{2}{33} \\ &= \frac{22}{33} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 7\frac{1}{5} \div 2\frac{2}{5} &= \frac{36}{5} \div \frac{12}{5} \\ &= \frac{36}{12} \\ &= 3 \end{aligned}$$

[2]
[2]

$$\begin{aligned} \text{c) } \sqrt{\frac{22}{5} \times \frac{165}{6}} &= \sqrt{\frac{11}{5} \times \frac{165}{2}} \\ &= \sqrt{\frac{11}{5} \times \frac{33}{1}} \\ &= \sqrt{\frac{11}{1} \times \frac{11}{1}} \\ &= 11 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{5}{2 - \frac{1}{1 - \frac{1}{3}}} &= \frac{5}{2 - \frac{1}{\frac{2}{3}}} \\ &= \frac{5}{2 - \frac{3}{2}} \\ &= \frac{5}{\frac{1}{2}} \\ &= 10 \end{aligned}$$

[2]
[3]

3. a) $p=3$, $q=12$ and $r=-5$.

Evaluate $p\sqrt{q^2+r^2}$.

$$= 3\sqrt{144+25}$$

$$= 3\sqrt{169}$$

$$= 3 \times 13$$

$$= 39$$

b) $2(x+3)-(x-5)+4(x+2)=119$. Find x .

$$2x+6-x+5+4x+8=119$$

$$5x=100$$

$$x=20$$

[2]
[2]

c) $\frac{c^3-14}{5}=10$. Find c .

$$c^3-14=50$$

$$c^3=64$$

$$c=4$$

d) $\frac{72}{y-21}=8$. Find y .

$$\frac{72}{8}=y-21$$

$$y-21=9$$

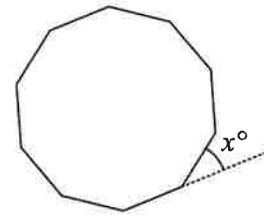
$$y=30$$

[2]
[2]

4. a) The diagram below shows a regular decagon (ten equal sides with ten equal interior angles). Write down the value of x .

[1]

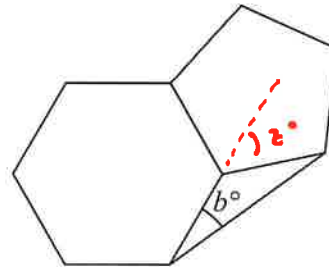
$$x = 36$$



- b) The diagram below shows a regular hexagon and a regular pentagon (which share a side). Find the value of b .

[3]

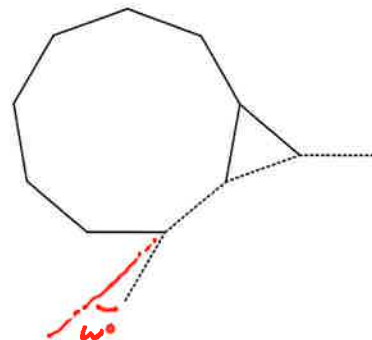
$$\begin{aligned} z &= 120 + 108 - 180 \\ &= 48 \\ b &= z \div 2 \\ &= 24 \end{aligned}$$



- c) The diagram below shows a regular nonagon (nine sides), an equilateral triangle and part of a regular polygon (dotted) with n sides. Find n .

[3]

$$\begin{aligned} w &= 60 - \frac{360}{9} \\ &= 20 \\ n &= \frac{360}{20} \\ &= 18 \end{aligned}$$



5. a) Find the value of $\frac{16^3}{2^9}$.

$$\begin{aligned} &= \frac{(2^4)^3}{2^9} \\ &= \frac{2^{12}}{2^9} \\ &= 2^3 \\ &= 8 \end{aligned}$$

b) $10^a \times 10^b = 10^7$.
 $10^a \div 10^b = 10^3$.

Find the value of ab .

$$\begin{aligned} a + b &= 7 \\ a - b &= 3 \\ a &= 5 \\ b &= 2 \\ ab &= 10 \end{aligned}$$

[2]
[2]

c) By writing each number as a product of prime factors, evaluate $\sqrt{15 \times 21 \times 35}$.

$$\begin{aligned} &= \sqrt{3 \times 5 \times 3 \times 7 \times 5 \times 7} \\ &= \sqrt{3^2 \times 5^2 \times 7^2} \\ &= 3 \times 5 \times 7 \\ &= 105 \end{aligned}$$

d) By writing 2020 as a product of prime factors, find a three-digit number n for which $2020n$ is a square number.

$$\begin{aligned} 2020 &= 2^2 \times 5 \times 101 \\ n &= 5 \times 101 \\ &= 505 \end{aligned}$$

[2]
[3]

6. a) Alice, Brenda and Clara share a pile of buttons in the ratio 7:12:21. Clara gets ten more buttons than Alice and Brenda put together. How many buttons were in the pile?

[2]

$$21 - (7 + 12) = 2$$

$$\frac{10}{2} = 5$$

$$5 \times (7 + 12 + 21) = 200$$

- b) Derek, Ethan, Fynn and Gerald share a big pile of sweets. Derek, Ethan and Fynn now have sweets in the ratio 1:4:6, and Ethan, Fynn and Gerald have sweets in the ratio 6:9:2. Derek and Gerald have 35 sweets between them. How many sweets does Gerald have?

[3]

$$\begin{array}{cccc} D & E & F & G \\ & 1 & : & 4 & : & 6 \\ \times) \left(& & & 6 & : & 9 & : & 2 \right) \times 2 \\ & & & 3 & : & 12 & : & 18 & : & 4 \end{array}$$

$$\frac{35}{3+4} = 5, \text{ so } G \text{ has } 20 \text{ sweets}$$

- c) Harriet and Imogen share sweets in the ratio 1:2. Imogen gives Harriet 42 sweets. The sweets are now shared between Harriet and Imogen in the ratio 5:4. How many sweets did they share?

[4]

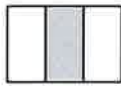
$$\begin{array}{cc} H & I \\ 1 & : & 2 \\ \rightarrow & 5 & : & 4 \end{array}$$

$$\frac{42}{2} = 21$$

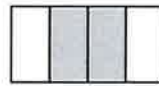
$$21 \times 9 = 189$$

7. a)

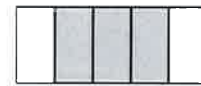
Strip Pattern 1



Strip Pattern 2



Strip Pattern 3



Strip Pattern number	1	2	3	4	5	6	n
Fraction of strip that is shaded	$\frac{1}{3}$	$\frac{2}{4}$	$\frac{3}{5}$	$\frac{4}{6}$	$\frac{5}{7}$	$\frac{6}{8}$	$\frac{n}{n+2}$

(i) Complete the table above.

[2]

(ii) Strip pattern k is 90% shaded. Find k .

[2]

$$\frac{k}{k+2} = \frac{9}{10}$$

$$10k = 9k + 18$$

$$k = 18$$

b) Complete the table below (a match is one side of a small square).

[3]

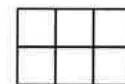
Rectangle Pattern 1



Rectangle Pattern 2



Rectangle Pattern 3

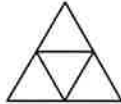


Rectangle Pattern number	1	2	3	4	11	n
Number of matches	7	12	17	22	57	$5n + 2$

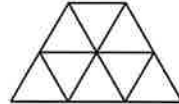
c) Find a formula for the number of matches in trapezium pattern n (a match is one side of a small equilateral triangle).

[2]

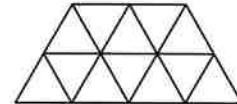
Trapezium Pattern 1



Trapezium Pattern 2



Trapezium Pattern 3



$$= 7n + 2$$

d) Frank makes a rectangle pattern, using all the matches he has. He then rearranges these matches into a trapezium pattern and has no matches left. Find three possible values for the number of matches Frank started with.

[3]

$$5n + 2 = 7m + 2$$

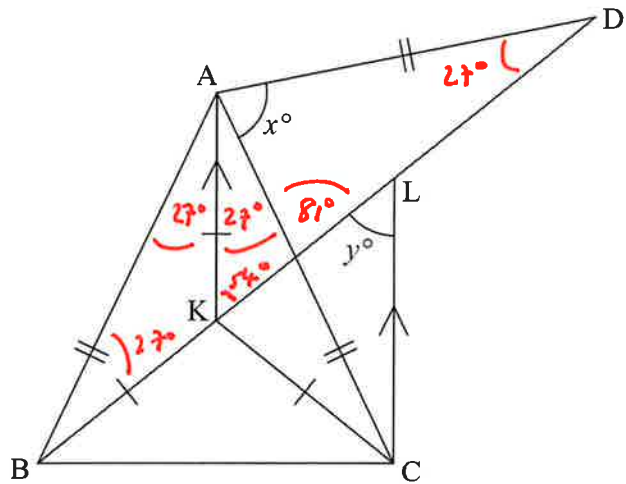
$$5n = 7m$$

The number of matches is a multiple of 35, plus 2

Three possible values are 37, 72 and 107.

8. a) In the diagram below $AB = AC = AD$, $KA = KB = KC$, KA is parallel to CL , and BKD is a straight line.

Angle $BAC = 54^\circ$. Find x and y .



[5]

$$x = 72$$

$$y = 54$$

- b) $ABCD$ is a rhombus and KLC is an equilateral triangle. The lengths of the sides of KLC are the same as the lengths of the sides of $ABCD$. Find z .

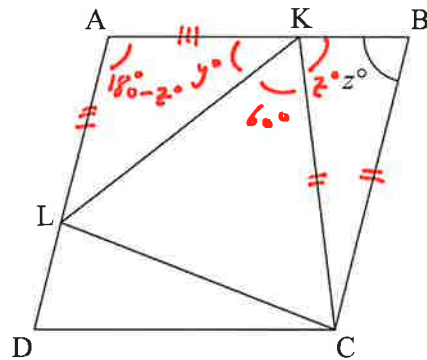
[5]

$$y = \frac{z}{2}$$

$$z + 60 + y = 180$$

$$z + \frac{z}{2} = 120$$

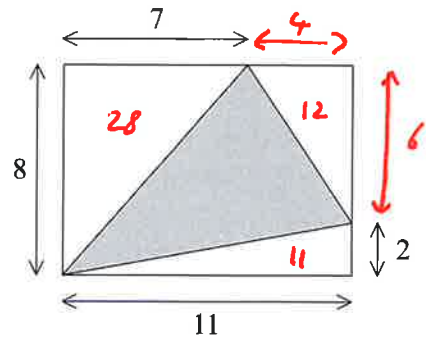
$$z = 80$$



9. a) The diagram shows a triangle drawn inside a rectangle. Find the shaded area.

[3]

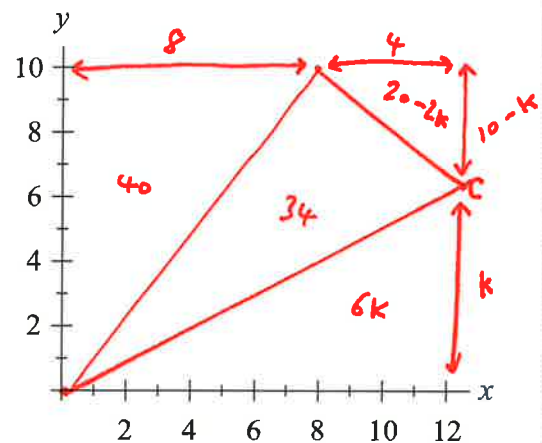
$$\begin{aligned} \text{Shaded area} &= 88 - 28 - 12 - 11 \\ &= 37 \end{aligned}$$



- b) The vertices of a triangle ABC are A(0, 0), B(8, 10) and C(12, k).

The area of the triangle ABC is 34.

- (i) Plot the points A and B on the diagram.
 (ii) Find the value of k , given that $0 < k < 10$.



[6]

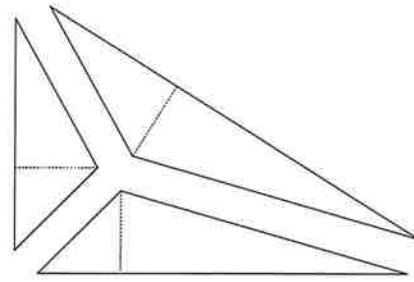
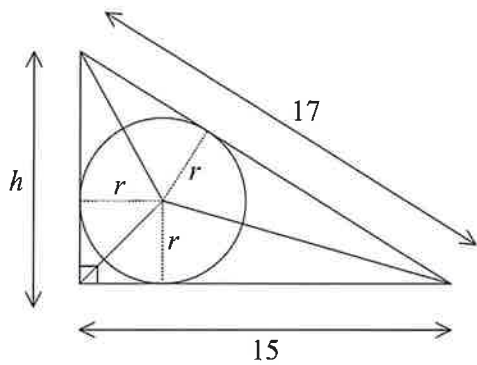
$$40 + 34 + (20 - 2k) + 6k = 120$$

$$94 + 4k = 120$$

$$4k = 26$$

$$k = 6.5$$

10. The diagram on the left below shows a right-angled triangle with its inscribed circle, radius r .



- a) Find h , and hence the perimeter and the area of the triangle.

[3]

$$h^2 = 17^2 - 15^2$$

$$h = 8$$

$$P = 8 + 15 + 17$$

$$= 40$$

$$A = \frac{1}{2} \times 15 \times 8$$

$$= 60$$

- b) By considering the three triangles in the diagram on the right above, find the value of r .

[3]

$$A = \frac{1}{2} \times r \times 15 + \frac{1}{2} \times r \times 8 + \frac{1}{2} \times r \times 17$$

$$A = \frac{1}{2} r \times P$$

$$60 = \frac{1}{2} \times 40 \times r$$

$$r = 3$$

ABCD is a tetrahedron with a horizontal base and one vertical edge. Note that the base is an isosceles triangle.

c) Find the volume and the total surface area of the tetrahedron.

[7]

(The volume of a pyramid is equal to $\frac{1}{3} \times \text{base area} \times \text{height}$.)

$$V = \frac{1}{3} \times (9 \times 12) \times 9$$

$$= 324$$

$$A = 2 \times \frac{1}{2} \times 15 \times 9 \quad (\triangle BCD + \triangle ACD)$$

$$+ \frac{1}{2} \times 18 \times 12 \quad (\triangle ABC)$$

$$+ \frac{1}{2} \times 18 \times 15 \quad (\triangle ABD)$$

$$= 135 + 108 + 135$$

$$= 378$$

d) Hence find the radius of the largest sphere that can be placed entirely inside the tetrahedron. Give your answer as a fraction in its simplest terms.

[4]

$$V = \frac{1}{3} r \times A$$

$$324 = \frac{1}{3} \times 378 \times r$$

$$126r = 324$$

$$r = \frac{324}{126}$$

$$= \frac{18}{7}$$

(END OF PAPER)