Name ......APM..

WINCHESTER

COLLEGE

ELECTION

Mathematics 1

Monday 23 April 2018

Time allowed: 1 hour 30 minutes

Total marks: 100

Calculators are not allowed.

Write your answers in this booklet. If you need additional space, please write on sheets of A4 paper and attach them to this booklet. You may use a pencil for diagrams.

Work carefully, and *do not be discouraged if you do not finish*.

You should show your working so that credit may be given for partly correct answers.

1. Evaluate:

a) $\dfrac{8072}{4}$ $= 2018$

b) $(4+7)(8-5)^2$ $= 11 \times 3^2$

$= 99$

[1]
[1]

c) $2^7 \div 4^3$

$= 2^7 \div 2^6$

$= 2$

d) $\dfrac{32100+6420+963}{321}$

$= 100 + 20 + 3$

$= 123$

[2]
[2]

e) $\dfrac{0.4\times 0.09}{0.0012}$

$= \dfrac{40 \times 9}{12}$

$= 30$

f) $\sqrt[3]{-125}$

$= -5$

[2]
[2]

2.	Evaluate, giving your answer in the simplest form: a) $\frac{3}{8}-\frac{5}{24}$ $= \frac{9}{24} - \frac{5}{24}$ $= \frac{4}{24}$ $= 1/6$	b) $2\frac{1}{4}\times 2\frac{2}{3}$ $= 9/4 \times 8/3$ $= 6$	[2] [2]
	c) $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}$ $= \dfrac{4+3}{4-3}$ $= 7$	d) $(\sqrt[3]{9})^6$ $= 9^2$ $= 81$	[2] [2]
	e) $\sqrt{0.1}\times\sqrt{100000}$ $= \sqrt{10000}$ $= 100$	f) $\dfrac{0.\dot{3}\times 0.\dot{6}}{0.\dot{2}}$ $(0.\dot{3}=0.333\ \ldots\ .)$ $= \dfrac{0.\dot{2}}{0.\dot{2}}$ $= 1$	[2] [2]

3. a) $\sqrt[3]{222-\dfrac{36}{a}}=6$. Find a. [2]

$222 - \frac{36}{a} = 216$

$\frac{36}{a} = 6$

$a = 6$

b) $\dfrac{1313}{10+\dfrac{99}{b}}=101$. Find b. [2]

$10 + \frac{99}{b} = 13$

$\frac{99}{b} = 3$

$b = 33$

c) $\dfrac{1}{4c-9}=\dfrac{2}{2c+3}$. Find c. [3]

$\frac{2}{8c-18} = \frac{2}{2c+3}$

$8c-18 = 2c+3$

$6c = 21$

$c = 7/2$

d) $\dfrac{8}{\sqrt{d}}=\dfrac{d}{8}$. Find d. [3]

$(64 = d\sqrt{d})$

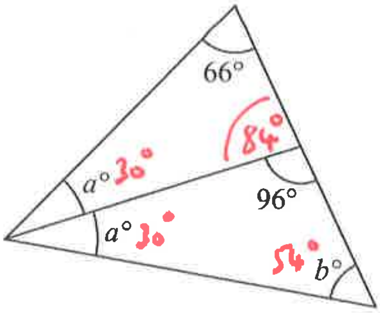
$(2^6 = d^{3/2})$

$(d = 2^4)$

$d = 16$

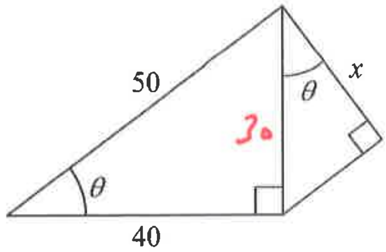
4.

a) In the diagram below, lines that look straight are straight. Find b.



$b = 54$

b) Find x.

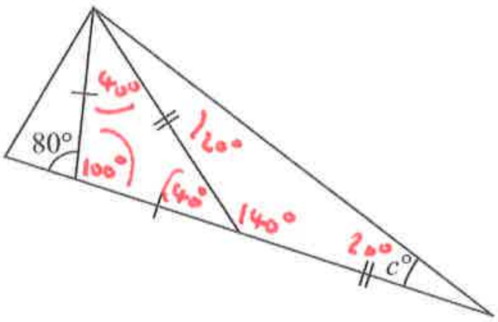


$x = \frac{4}{5} \times 30$

$= 24$

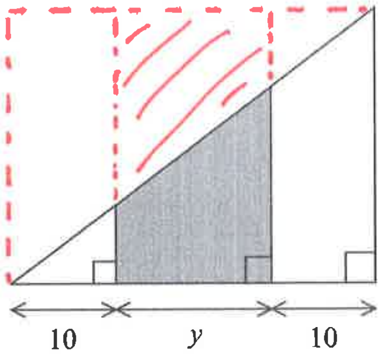
[2]
[3]

c) In the diagram below, lines that look straight are straight. Find c.



$c = 20$

d) Three-eighths of the triangle is shaded. Find y.



$y = \frac{3}{8}(20 + y)$

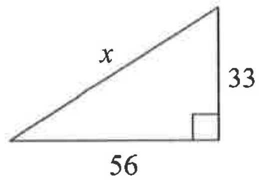
$8y = 60 + 3y$

$5y = 60$

$y = 12$

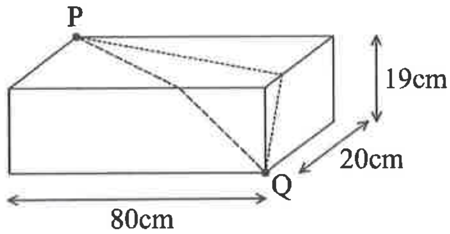
[3]
[4]

5. a) Use the table of squares at the bottom of the opposite page to find x. [2]

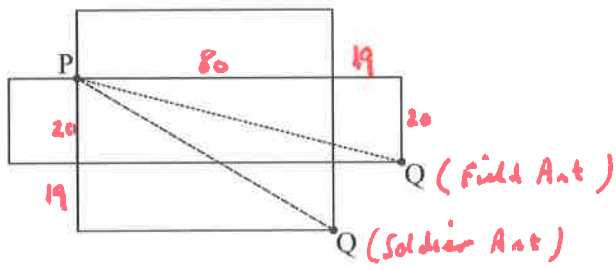


$x^2 = 33^2 + 56^2$
$= 1089 + 3136$
$= 4225$
$x = 65$

The diagram below shows a cuboid made out of thin card. A soldier ant and a field ant are sitting at the point P, arguing about the shortest route to Q. The soldier ant has decided to go via the top front edge (dashed route), and the field ant has decided that he will go via the top right edge (dotted route).



A leaf cutter ant crawls up to them. He helpfully suggests that, were he to cut the box and flatten it (see below), each ant might then see how to work out the length of its shortest route.



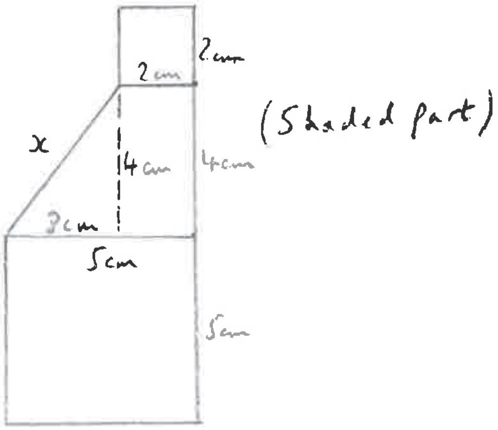
b) Find the length of the shortest route for each ant. [5]

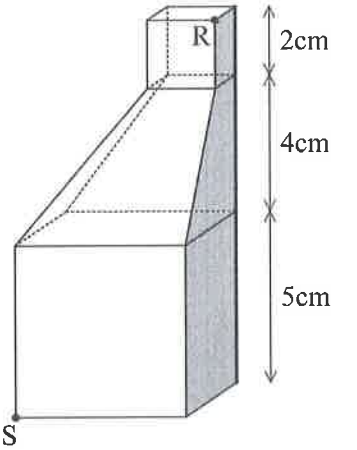
Soldier Ant : $d^2 = 80^2 + 39^2$
$= 6400 + 1521$
$= 7921$
$= 89^2$
$d = 89$

Field Ant : $d^2 = 99^2 + 20^2$
$= 9801 + 400$
$= 10201$
$d = 101$

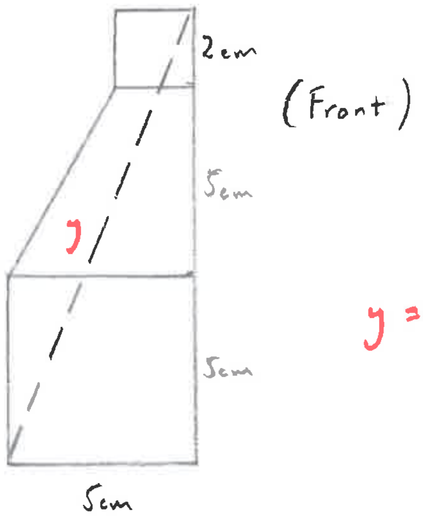
c) The diagram shows a model building comprising a $2\text{cm} \times 2\text{cm} \times 2\text{cm}$ cube and a $5\text{cm} \times 5\text{cm} \times 5\text{cm}$ cube joined together by four edges. The bold line is straight and the shaded part is flat.

A yellow crazy ant (*Anoplolepis gracilipes*) wants to crawl from point R to point S. Find the length of its shortest possible route. [6]





$x = 5\text{cm}$, by pythagoras.

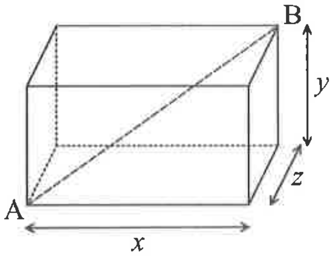


$y = 13\text{cm}$, by pythagoras.

Table of squares. Example: $73^2 = (70+3)^2 = 5329$.

	0	1	2	3	4	5	6	7	8	9
0	0	1	4	9	16	25	36	49	64	81
10	100	121	144	169	196	225	256	289	324	361
20	400	441	484	529	576	625	676	729	784	841
30	900	961	1024	1089	1156	1225	1296	1369	1444	1521
40	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
50	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
60	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
70	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
80	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
90	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

6. The diagram below shows a cuboid with the long diagonal AB. If the length of this long diagonal is d, then $d=\sqrt{x^2+y^2+z^2}$. (This is Pythagoras in three dimensions.)



a) Find the value of d if $x=6$, $y=3$ and $z=2$. [1]

$d = \sqrt{6^2+3^2+2^2}$

$= \sqrt{49}$

$= 7$

b) Complete the statements below:

Surface area of the cuboid $=2xy+$ $2xz + 2yz$;

Sum of the lengths of the edges of the cuboid $=4x+$ $4y + 4z$. [2]

The diagram below shows a square divided up into nine rectangles. (Note that a square is a special kind of rectangle.) The areas of four of the rectangles are given.

	x	y	z
x	x^2	xy	xz
y	xy	y^2	yz
z	xz	yz	z^2

c) Write in the areas of the remaining five rectangles, and then complete the statement below:

Area of big square $=(x+y+z)^2=x^2+$ y^2+z^2 $+2xy+$ $2xz+2yz$. [2]

d) The length of the long diagonal of a cuboid is 15, and the sum of the lengths of the edges is 84. Find the surface area of the cuboid. [5]

$x^2+y^2+z^2=15^2$

$(=225)$

$x+y+z = 84/4$

$= 21$

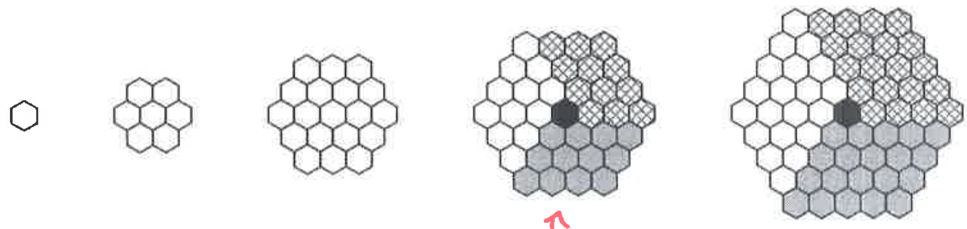
$2xy+2xz+2yz = (x+y+z)^2 - x^2+y^2+z^2$

$= 21^2-15^2$

$= 441-225$ (or $(21-15)(21+15)$)

$= 216$

7. An n-honeycomb is a hexagonal array of regular hexagonal cells which has n of these hexagonal cells along each edge. The length of a side of any cell is 1 unit. The diagram below shows the n-honeycomb for $n = 1$, 2, 3, 4 and 5. (Some cells are shaded to help you count them.)



4×3 5×4

a) How many cells are there in a 6-honeycomb? [1]

$3 \times (6 \times 5) + 1 = 91$

b) How many cells are there in a 10-honeycomb? [1]

$3 \times (10 \times 9) + 1 = 271$

c) There are 30907 cells in a k-honeycomb. Find k. [3]

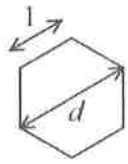
$3k(k-1) + 1 = 30907$

$3k(k-1) = 30906$

$k(k-1) = 10302$

$k = 102$

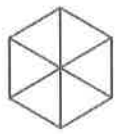
d) Eight points are placed randomly in a 2-honeycomb. Explain why there must be a pair of these points that are no farther than two units apart. (*Hint*: what is d in the diagram below?) [2]



$d = 2$

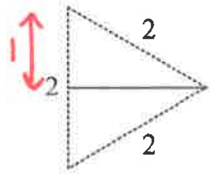
THERE ARE SEVEN HEXAGONS AND EIGHT POINTS. SO THERE MUST BE A HEXAGON WITH MORE THAN ONE POINT IN IT. CONSIDER TWO POINTS IN THIS HEXAGON. THEY CANNOT BE MORE THAN d (ie 2 UNITS) APART.

e) 115 points are placed randomly in the 3-honeycomb. Explain why there must be a pair of these points that are no farther than one unit apart. (*Hint*: look at the diagram below.) [3]



THERE ARE 19 HEXAGONS IN THE 3-HONEYCOMB. SO THERE ARE $19 \times 6 = 114$ EQUILATERAL TRIANGLES OF SIDE LENGTH 1 UNIT IN THE 3-HONEYCOMB. THERE ARE 115 POINTS, SO THERE MUST BE AN EQUILATERAL TRIANGLE WITH MORE THAN ONE POINT IN IT. CONSIDER TWO POINTS IN THIS TRIANGLE. THEY CANNOT BE MORE THAN 1 UNIT APART (BECAUSE THE EQUILATERAL TRIANGLE HAS SIDE LENGTH 1.)

8.	a) In the diagram on the right, the length of the bold line is $\sqrt{k}$. Find k.	[1]

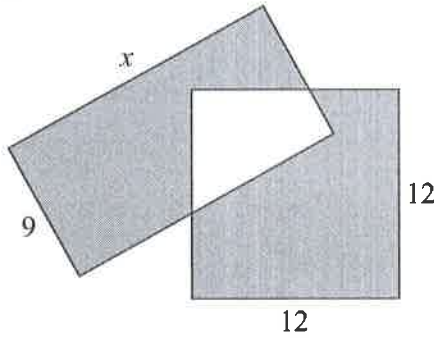


$1^2 + (\sqrt{k})^2 = 2^2$

$1 + k = 4$

$k = 3$

b) In the diagram below, the two shaded areas are equal. Find x. [3]



SHADED AREA OF RECTANGLE + OVERLAP = $9x$

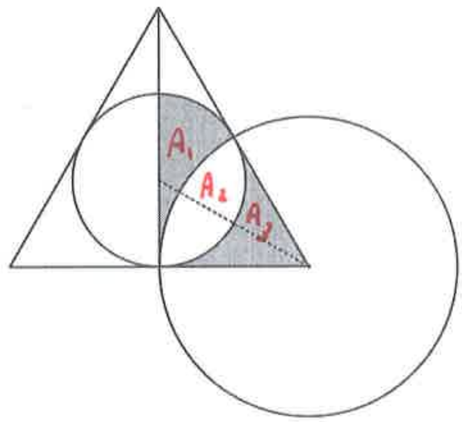
SHADED AREA OF SQUARE + OVERLAP = 12×12

SO $9x = 144$

$x = 16$

In the diagram below, the centre of the big circle is a vertex of the equilateral triangle and the smaller circle touches each of the three sides of the equilateral triangle. The vertical line is a tangent to the big circle and cuts the equilateral triangle in half. The radius of the smaller circle is one.

c) Prove that the two shaded areas are equal. (*Hint*: you might find the dotted line useful.) [7]



$A_1 + A_2 = \frac{1}{2} \times \pi \times 1^2$

$= \frac{1}{2}\pi$

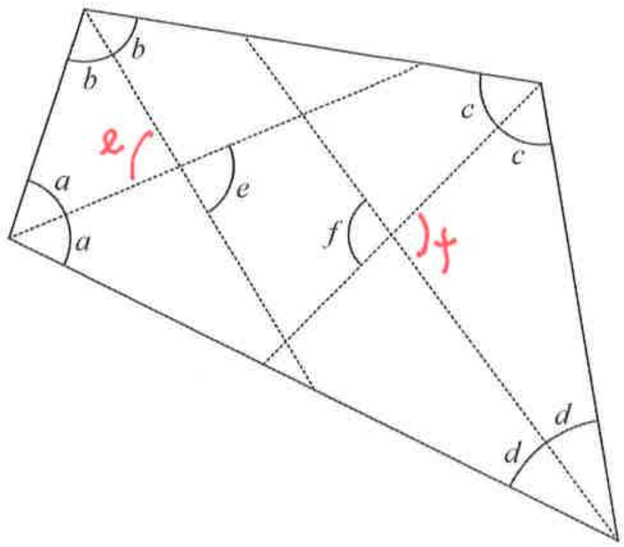
$A_2 + A_3 = \frac{1}{6} \times \pi \times (\sqrt{3})^2$

$= \frac{1}{2}\pi$

So $A_1 + A_2 = A_2 + A_3$

i.e. $A_1 = A_3$

9. a)



The diagram shows a quadrilateral with its angle bisectors drawn inside. Prove that $e + f = 180°$. [5]

$e = 180° - a - b$

$f = 180° - c - d$

$e + f = 360° - (a + b + c + d)$

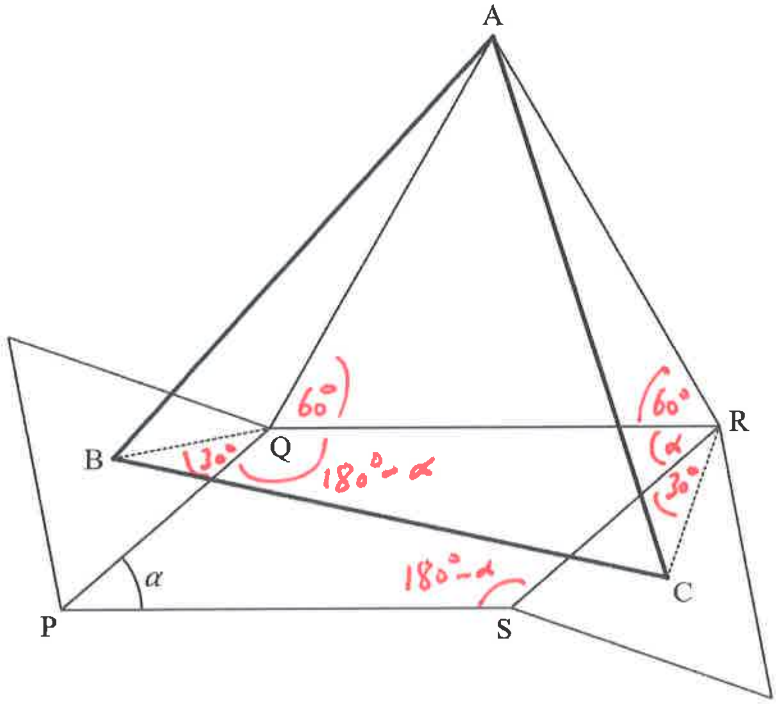
$= 360° - \frac{1}{2}(2a + 2b + 2c + 2d)$

$= 360° - \frac{1}{2} \times 360°$

(ANGLES IN A QUADRILATERAL SUM TO 360°)

$= 180°$

b)



The diagram shows a parallelogram PQRS with equilateral triangles on three of its sides. B is the centre of the equilateral triangle on PQ, and C is the centre of the equilateral triangle on SR. Prove that ABC is equilateral. [7]

$\angle BQA = 360° - (30° + 180° - \alpha + 60°)$

$= 360° - (270° - \alpha)$

$= 90° + \alpha$

$\angle ARC = 60° + \alpha + 30°$

$= 90° + \alpha$

So $\angle BQA = \angle ARC$

AQ = AR (EQ. TRIANGLE)

BQ = CR (EQUAL EQ. TRIANGLES)

So BQA & ARC ARE CONGRUENT.

$\therefore$ AB = AC

$\angle BAQ = \angle CAR$ (CONGRUENT TRIANGLES)

$\angle QAR = 60°$

So $\angle BAC = 60°$

$\therefore$ ABC IS EQUILATERAL